

KNOWLEDGE INSTITUTE OF TECHNOLOGY
(Approved by AICTE, New Delhi and Affiliated to Anna University, Chennai)
Kakapalayam (PO), Salem – 637504.

Department of Mechanical Engineering



Beyond Knowledge

COURSE PLAN

FINITE ELEMENT ANALYSIS

Department of Mechanical Engineering

**FINITE ELEMENT ANALYSIS
TWO MARK QUESTIONS**

UNIT 1- INTRODUCTION**1. Name the variational methods.**

1. Ritz method
2. Rayleigh –Ritz method

2. Name the weighted residual methods.

- Point collocation method
- Sub-domain collocation method
- Least squares method
- Galerkin's method.

3. What is Raleigh – Ritz method?

Rayleigh-Ritz method is an integral method which is useful for solving complex structural problems, encountered in finite element analysis. This method is possible only if a suitable functional is available.

4. What is meant by discretization and assembly?

The art of sub dividing the structure into a convenient number of smaller components is known as discretisation. The smaller components are put together and this process of combining all the elements together is known as assemblage.

5. What is aspects ratio?

Aspect ratio is the ratio of the largest dimension of the element to the smallest dimension of the element. In many cases, if the aspect ratio increases the inaccuracy of the solution increases. The aspect ratio should be close to unity as for as possible.

6. What is meant by finite element analysis?

Finite element methods is a numerical method for solving problems of engineering and mathematical physics.

In this method, instead of solving the problem for the entire body in one operation, we formulate equations for each element and combine them to obtain the solution for the whole body.

7. What are the types of boundary condition?

There are two types of boundary condition. They are:

1. Primary boundary condition
2. Secondary boundary condition

8. What are the methods generally associated with the finite element analysis?

Force method and Displacement or stiffness method are the two methods.

9. Explain force method.

In force method, internal forces are considered as unknowns of the problem. In Displacement or stiffness method, the displacements are considered as unknowns of the problem. Among the two methods, displacement method is desirable.

10. Why polynomial type of interpolation functions are mostly used due to the following Reasons:

1. It is easy to formulate and computerize the finite element equations
2. It is easy to perform differentiation or integration
3. The accuracy of the results can be improved by increasing the order of the polynomial.

11. What are 'h' and 'p' versions of finite element method?

In 'h' version, the order of the polynomial approximation for all elements is kept constant and the number of elements increased.

In 'p' version the number of elements is maintained constant and the order of polynomial approximation of element is increased.

12. Name any four FEA software's.

1. Ansys
2. NASTRAN
3. COSMOS
4. NISA

13. Differentiate between global and local axes.

Local axes are established in an element, they change with change in orientation of the element. The direction differs from element to element.

Global axes are defined for the entire system. They have the same direction for all the elements even though the elements are differently oriented.

UNIT 2- ONE-DIMENSIONAL PROBLEMS**1. What are the types loading acting on a structure?**

There are three type of loading acting on a structure. They are,

- a. Body force (f)
- b. Traction force (T)
- c. Point load (P)

2. Define body force.

A body force is distributed force acting on every elemental volume of the body.

Unit: force per unit volume

3. Define traction force.

Traction force is defined as distributed force acting on the surface of the body.

Unit: Force per unit area

Examples: Frictional resistance, viscous

4. What is a point load?

Point load is load acting at a particular point which causes displacement.

5. What are the basic steps involved in the finite element modeling.

Finite element modeling consists of the following:

1. Discretisation of the structure
2. Numbering of the nodes.

6. What are the classifications of the co-ordinates?

The co-ordinates are generally classified as ,

1. Global co-coordinates
2. Local Co-ordinates
3. Natural co-ordinates

7. What is natural co-ordinates?

A natural co-ordinate system is used to define any point inside the element by a set of dimensionless numbers, whose magnitude never exceeds unity. This system is useful in assembling of stiffness matrices.

8. Define shape function.

In finite element method, field variables within an element are generally expressed by the following approximate relation: $(x,y) = N_1(x,y)_1 + N_2(x,y)_2 + N_3(x,y)_3$ where $1\ 2\ 3\ 4$ are the values of the field variable at the nodes and $N_1\ N_2\ N_3\ N_4$ are interpolation function. $N_1\ N_2\ N_3\ N_4$ are called shape functions because they are used to express the geometry or shape of the element.

9. What are the characteristics of shape function?

The characteristics of the shape functions are follows:

1. The shape function has unit value at one nodal point and zero value at the other nodes.

2. The sum of the shape function is equal to one.

10. Why polynomials are generally used as shape function?

Polynomials are generally used as shape functions due to the following reasons:

1. Differentiation and integration of polynomials are quite easy.
2. The accuracy of the results can be improved by increasing the order of the polynomial.
3. It is easy to formulate and computerize the finite element equations.

11. Give the expression for element stiffness matrix.

Stiffness matrix $[K] = [B] [D][B]^T v$

Where, $[B]$ matrix is a strain displacement matrix

$[D]$ matrix is stress, strain relationship matrix

12. Write down the expression of stiffness matrix for one dimensional element bar element.

$$\text{Stiffness matrix } [K] = \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Where, A is the area of the bar element

E is the young's modulus of the bar element

L is the length of the bar element

13. State the properties of a stiffness matrix.

The properties of the stiffness matrix $[K]$ are,

1. It is a symmetric matrix
2. The sum of the elements in any column must be equal to zero.
3. It is an unstable element, so the determinant is equal to zero.

14. Write down the general finite element equation.

General finite element equation is,

$$\{F\} = [K] \{u\}$$

Where, $\{F\}$ is a force vector

$[K]$ is the stiffness matrix

{u} is the degrees of freedom

15. State the assumptions made in the case of truss element.

The following assumptions are made in the case of truss element,

1. All the members are pin jointed.
2. The truss is loaded only at the joints
3. The self weight of the members are neglected unless stated.

16. Write down the expression of stiffness matrix for a truss element.

$$\text{Stiffness matrix, } [K] = \frac{A_e E_e}{l_e} \begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

Where,
 A is the area
 E is the young's modulus
 L_e is the length of the element
 L, m are direction cosines

17. Write down the expression of shape function N and displacement u for one dimensional bar element.

For one dimensional bar element,

$$\text{Displacement function, } u = N_1 u_1 + N_2 u_2$$

Where, Shape function, $N_1 = \frac{l-x}{l}$

$$N_2 = \frac{x}{l}$$

18. State the principle of minimum potential energy.

The total potential energy of an elastic body is defined as the sum of total strain energy U and the potential energy of the external forces, (W)

19. Distinguish between essential boundary condition and natural boundary condition.

There are two types of boundary conditions. They are,

1. Primary boundary condition (or) essential boundary condition:

The boundary condition which in terms of the field variables is known as primary boundary condition

2. Secondary boundary condition or natural boundary condition:

The boundary conditions which are in the differential form of field variables is known as secondary boundary condition.

20. What are the difference between boundary value problem and initial value problem?

The solution of differential equation obtained for physical problems which satisfies some specified conditions known as boundary conditions.

If the solution of differential equation is obtained together with initial conditions then it is known as initial value problem. If the solution of differential equation is obtained together with boundary conditions then it is known as boundary value problem.

UNIT 3- TWO DIMENSIONAL SCALAR VARIABLE PROBLEMS

1. What is a CST element?

Three noded triangular element is known as constant strain triangular element. It has 6 unknown degrees of freedom called $u_1, v_1, u_2, v_2, u_3, v_3$. The element is called CST because it has constant strain throughout it.

2. What is LST element?

Six noded triangular element is known as Linear Strain Triangular element. It has 12 unknown displacement degrees of freedom. The displacement function for the element are quadratic instead of linear as in the CST.

3. What is a QST element?

Ten noded triangular element is known as Quadratic Strain Triangle.

4. What is meant by plane stress analysis?

Plane stress is defined as a state of stress in which the normal stress (σ_z) and the shear stress (τ_{xz}, τ_{yz}) directed perpendicular to the plane are zero.

5. Define plane strain.

Plane strain is defined to be a state of strain in which the strain normal to the xy plane and the shear strains are assumed to be zero.

6. Write the displacement function for a CST element.

$$\text{Displacement function } u = \begin{Bmatrix} u(x,y) \\ v(x,y) \end{Bmatrix} = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix} \text{ where } N_i$$

N_1, N_2, N_3 are shape functions.

7. Write a strain displacement matrix for CST element.

Strain displacement equation for CST element is,

$$[B] = \frac{1}{2A} \begin{bmatrix} q_1 & 0 & q_2 & 0 & q_3 & 0 \\ 0 & r_1 & 0 & r_2 & 0 & r_3 \\ r_1 & q_1 & r_2 & q_2 & r_3 & q_3 \end{bmatrix}$$

Where, A = Area of the element

$$q_1 = y_2 - y_3 \quad q_2 = y_3 - y_1 \quad q_3 = y_1 - y_2$$

$$r_1 = x_3 - x_2 \quad r_2 = x_1 - x_3 \quad r_3 = x_2 - x_1$$

8. Write down the stress –strain relationship matrix for plane stress condition.

For plane stress problems, stress –strain relationship matrix is ,

$$[D] = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

Where, E = young's modulus
 ν = Poisson's ratio

9. Write down the stress-strain relationship matrix for plane strain condition.

For plane strain problems, stress – strain relationship matrix is,

$$[D] = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} (1-\nu) & \nu & 0 \\ 0 & (1-\nu) & 0 \\ 0 & 0 & \frac{(1-2\nu)}{2} \end{bmatrix}$$

10. Write down the stiffness matrix equation for two dimensional CST element.

$$\text{Stiffness matrix } [K] = [B]^T [D] [B] A t$$

Where, [B] is the strain displacement matrix

[D] is the stress-strain matrix

A is the area of the element

't' is the thickness of the element

UNIT 4- TWO DIMENSIONAL VECTOR VARIABLE PROBLEMS**1. Define Quasi static response.**

When the excitations are varying slowly with time then it is called quasi static response.

2 Give the Lagrange's equations of motion.

The Lagrange's equations of motion in the independent generalized coordinates q is given by,

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}} \right) - \frac{\partial T}{\partial q} + \frac{\partial \pi}{\partial q} = F_q$$

3. What are the conditions for a problem to axi symmetric?

1. The problem domain must be symmetric about the axis of rotation.
2. All the boundary conditions must be symmetric about the axis of rotation.
3. All loading conditions must be symmetric about the axis of rotation.
3. What is HRZ Lumping scheme.

The essential idea in this scheme is to simply use only diagonal elements of the consistent mass matrix but to scale them in such a way that total mass of the element is preserved.

4. Give the consistent mass matrix for bar element:

$$[m]^e = \rho A \int_0^{\ell} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{bmatrix} [N_1 \quad N_2 \quad N_3 \quad N_4] dx$$

5. State the methods of solution to eigen value problems.

There are essentially three groups of methods of solution of eigen value problems.

- i) Determinant based methods
- ii) Transformation based method
- iii) Vector iteration based method

6. Give the stiffness matrix equation for an axi-symmetric triangular element.

$$[K] = 2\pi r A [B]^T [D] [B]$$

$$r = \frac{r_1 + r_2 + r_3}{3}$$

7. What are the ways in which a three dimensional problem can be reduced to a two dimensional approach.

1. Plane Stress: one dimension is too small when compared to other two dimensions. Example: Gear – thickness is small
2. Plane Strain: one dimension is too large when compared to other two dimensions. Examples: Long Pipe (length is long compared to diameter)

UNIT 5- ISOPARAMETRIC FORMULATION**1. What is the purpose of Iso parametric elements?**

It is difficult to represent the curved boundaries by straight edges finite elements. A large number of finite elements may be used to obtain reasonable resemblance between original body and assemblage. In order to overcome this drawback, iso parametric elements are used i.e for problems involving curved boundaries, a family of elements known as 'iso parametric elements are used.

2. Write down the element level heat transfer equation for bar element with heat conduction.

$$\left(\frac{k}{\ell} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \right) \begin{Bmatrix} T_k \\ T_{k+1} \end{Bmatrix} = (q_0) \begin{Bmatrix} \ell/2 \\ \ell/2 \end{Bmatrix} + \begin{Bmatrix} -Q_o \\ Q_i \end{Bmatrix}$$

3. Write down the equation for the Jacobian matrix for four noded quadrilateral element.

Jacobian matrix, $[J] = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$

$$[J_{11}] = \frac{1}{4} [-(1-\eta)x_1 + (1-\eta)x_2 + (1+\eta)x_3 - (1+\eta)x_4]$$

$$[J_{12}] = \frac{1}{4} [-(1-\eta)y_1 + (1-\eta)y_2 + (1+\eta)y_3 - (1+\eta)y_4]$$

Where,

$$[J_{21}] = \frac{1}{4} [-(1-\epsilon)x_1 - (1+\epsilon)x_2 + (1+\epsilon)x_3 + (1-\epsilon)x_4]$$

$$[J_{22}] = \frac{1}{4} [-(1-\epsilon)y_1 - (1+\epsilon)y_2 + (1+\epsilon)y_3 + (1-\epsilon)y_4]$$

Where ϵ, η are natural co-ordinates.

$x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4$ are Cartesian co-ordinates.

4. Write down the stiffness matrix equation for four noded iso parametric quadrilateral element.

$$\text{Stiffness matrix, } [K] = t \int_{-1}^1 \int_{-1}^1 [B^T] [D] [B] |J| \partial \epsilon \partial \eta$$

Where, t is the thickness of the element

J is the determinant of the Jacobian., are natural co-ordinates

Matrix B is strain-Displacement matrix and matrix D is the stress-strain relationship matrix

5. Write down the element force vector equation for four noded quadrilateral element.

$$\text{Force vector, } \{F\}_e = [N]^T \begin{Bmatrix} F_x \\ F_y \end{Bmatrix}$$

Where N is the shape function

F_x, F_y are force on x, y direction respectively.

6. Write down the Gaussian quadrature expression for numerical integration.

Gaussian quadrature expression,

$$\int_{-1}^1 f(x) dx = \sum_{i=1}^n w_i f(x_i)$$

Where w_i is the weight function

$f(x_i)$ is the value of the function at pre determined sampling points.

7. Define super parametric element.

If the number of nodes for defining the geometry is more than the number of nodes used for defining the displacements is known as super parametric element.

8. What is meant by sub parametric element?

If the number of nodes used for defining the geometry is less than the number of noded used for defining the displacements is known as sub parametric element.

9. What is meant by iso parametric element?

If the number of nodes used for defining the geometry is same as number of nodes used for defining the displacements then it is called iso parametric element.

10. Is beam element an iso parametric element?

Beam element is not an iso parametric element since geometry and displacements are defined by different interpolation functions.

FEA QUESTION BANK
UNIT 1- INTRODUCTION

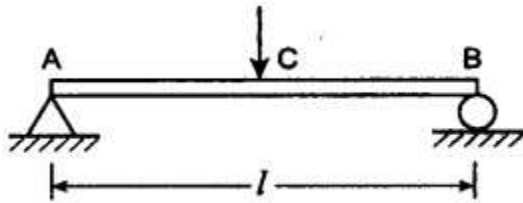
PART -A (2 Marks)

1. What is meant by finite element analysis?
2. Name any four applications of FEA.
3. What is the concept of matrix algebra and in what way it is used in FEA?
4. Briefly explain Gaussion elimination method.
5. Why polynomial type interpolation functions are preferred over trigonometric functions?
6. What is meant by 'descretization'?
7. List out the various weighted-residual methods.
8. Define the concept of potential energy
9. List out any four advantages of using FEA.
10. What is the need for FEA?
11. List out FEM software packages
12. Name the different modules of FEM and their function
13. What are 'h' and 'p' versions of Finite Element method?
14. Name the vibrational methods.
15. What is meant by post processing?
16. Distinguish between essential boundary conditions and natural boundary conditions.
17. What are the general constituents of finite element software?
18. What is Rayleigh-Ritz method?
19. During discretization, mention the places where it is necessary to place a node?.
20. What is meant by node?

UNIT I PART –B(16 Marks)

1. A simply supported beam is subjected to uniformly distributed load over entire span. Determine the bending moment and deflection at the mid span using Rayleigh-Ritz method and compare with exact solution. Use a two term trial function $y = a_1 \sin(\pi x/l) + a_2 \sin(3\pi x/l)$

2. A beam AB of span 'l' simply supported at the ends and carrying a concentrated load 'W' at the centre 'C' as shown in figure 1.2. Determine the deflection at the mid span by using Rayleigh-Ritz method and compare with exact solution. Use a suitable one term trigonometric trial function.



3. A simply supported beam is subjected to uniformly distributed load over entire span and it is subjected to a point load at the centre of the span. Calculate the bending moment and deflection at the mid span using Rayleigh-Ritz method and compare with exact solution.

4. The following differential equation is available for a physical phenomenon.

$$d^2y/dx^2 + 50 = 0, \quad 0 < x < 10$$

The trial function is, $y = ax(10-x)$

The boundary conditions are $y(0)=0$ and $y(10)=0$

Find the value of the parameter 'a' by (i) Point collocation method (ii) Sub-domain collocation method (iii) Least squares method (iv) Galerkin's method

5. Write short notes on (i) Gaussian elimination (ii) Galerkin's method

6. Explain the process of discretization in detail.

7. A cantilever beam of length 'L' is loaded with a point load at the free end. Find the maximum deflection and maximum bending moment using Rayleigh-Ritz method using the function $y = a\{1 - \cos(\pi x/2L)\}$. Given EI is constant.

8. Solve the simultaneous systems of equation using Gauss-Elimination method.

$$3x + y - z = 3 ; 2x - 8y + z = 3 ; x - 2y + 9z = 8$$

9. The following differential equation is available for a physical phenomenon:

$$d^2y/dx^2 - 10x^2 = 5 ; 0 = x = 1$$

The boundary conditions are: $y(0) = 0$ and $y(1) = 0$. By using Galerkin's method of weighted residuals, find an approximate solution of the above differential equation.

10. A simply supported beam subjected to uniformly distributed load 'q₀' over entire span and also point load of magnitude 'P' at the centre of the span. Calculate the bending moment and deflection at mid span by using Rayleigh-Ritz method and compare with the exact solution.

11. Solve the following equation using a 2 parameter trial solution by:

a) Point collocation method.

b) Galerkin's method.

$$dy/dx + y = 0, 0=x=1, y(0) = 1$$

12. The following differential equation is available for a physical phenomenon.

$d^2y/dx^2 + 50 = 0, 0=x=10$, Trial function is $y = a_1x(10-x)$, $y(0) = 0, y(10) = 0$. Find the value of a_1 using all the weighted residual techniques and compare the solutions.

13. Solve the given equations by Gauss-Elimination method.

$$2x + 4y + 2z = 15 ; 2x + y + 2z = -5 ; 4x + y - 2z = 0$$

14. List the advantages, disadvantages and applications of FEM.

15. Explain the various weighted residual techniques.

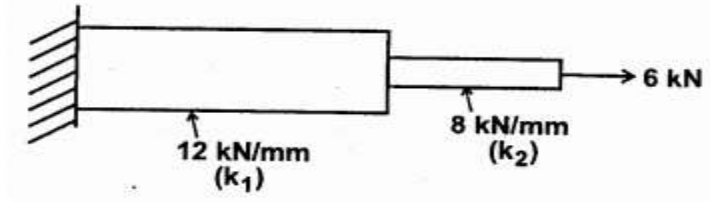
UNIT 2 - ONE -DIMENSIONAL PROBLEM**PART A**

1. List out the properties of stiffness matrix
2. What are the different coordinate systems used in FEM?
3. Define a simplex, complex and multiplex element
4. What are shape functions and what are their properties?
5. Define 'Natural coordinate system'
6. What are the advantages of natural coordinate system?
7. What are 1-Dimensional scalar and vector variable problems?
8. What types of problems are treated as one-dimensional problems?
9. Write down the expressions for shape functions of 1-D bar element.
10. Define aspect ratio. State its significance.
11. Write down the expressions for the element stiffness matrix of a beam element.
12. What is a higher order element? Give an example
13. Define shape function.
14. How do you calculate the size of the global stiffness matrix?
15. What are the characteristics of shape function?
16. What is truss?
17. Define total potential energy.
18. State the principle of minimum potential energy.
19. Write down the expression of shape function N and displacement u for 1-D bar element.
20. Write down the expression of stiffness matrix for a truss element.

UNIT 2 PART –B (16 Marks)

1. i) Derive the shape functions for a 2-D beam element
ii) Derive the stiffness matrix of a 2-D truss element

2. Derive the shape functions for a 2 noded beam element and a 3 noded bar element
3. Derive the stiffness matrix of a 3 noded bar element using the principle of potential energy
4. Calculate the nodal displacements and forces for the bar loaded as shown in figure.



5. A stepped bar is subjected to an axial load of 200 kN at the place of change of cross section and material as shown in figure 2.5. Find (a) The nodal displacements (b) the reaction forces (c) the induced stresses in each material

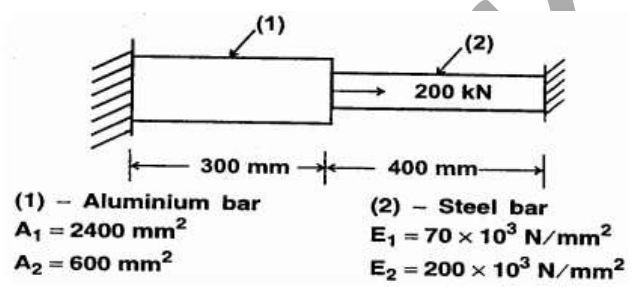
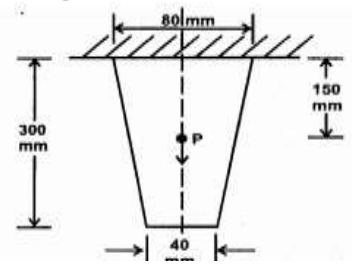


figure 2.5

6. For a tapered bar of uniform thickness $t=10\text{mm}$ as shown in figure 2.6. Find the displacements at the nodes by forming into two element model. The bar has a mass density $= 7800 \text{ Kg/M}$, young's modulus $E = 2 \times 10^5 \text{ MN/m}$. In addition to self-weight, the bar is subjected to a point load $P= 1 \text{ KN}$ at its centre. Also determine the reaction forces at the support.



7. Consider a 4-bar truss as shown in figure. It is given that $E = 200 \text{ GPa}$ and $A = 500 \text{ mm}^2$ for all the elements. Determine (a) Nodal displacements (b) Support reactions (c) Element stresses.

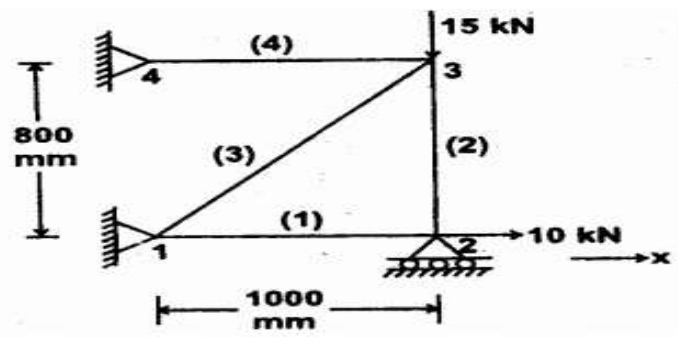
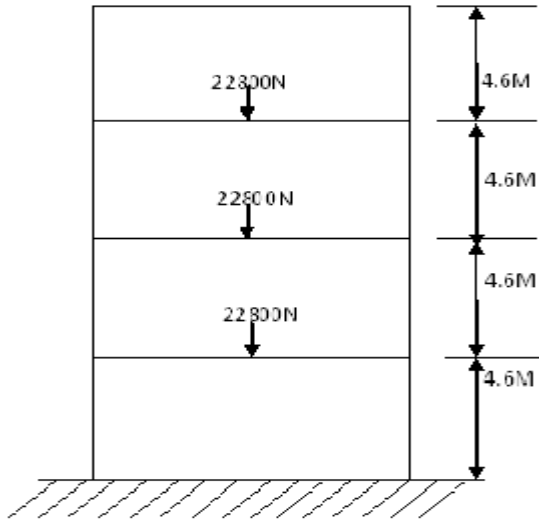
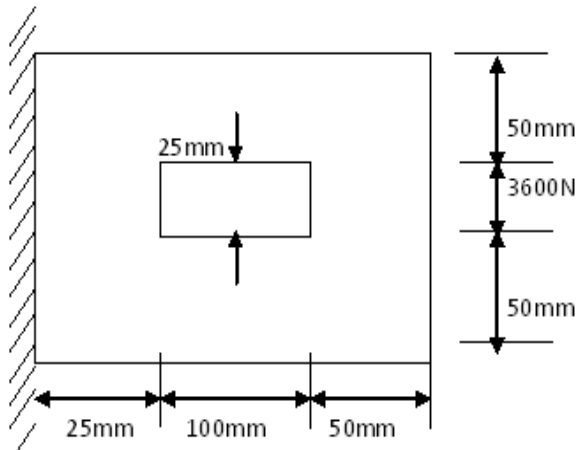


figure 2.7

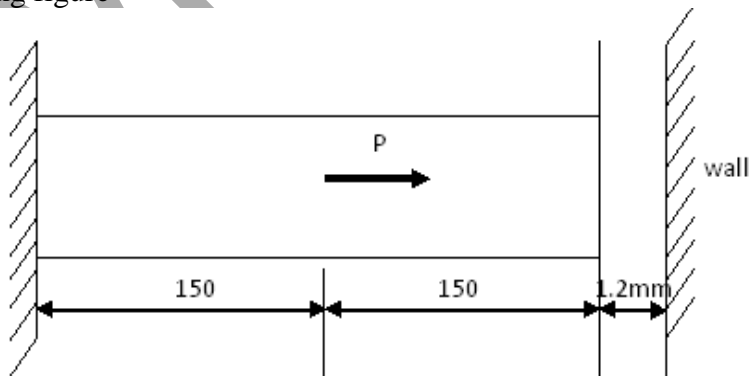
8. Derive an expression for shape function and assemble the stiffness matrix for bending in beam elements.
9. Derive an expression of shape functions and the stiffness matrix for one dimensional bar element based on global co-ordinate approach.
10. A two noded truss element is shown in the fig. The nodal displacements are $u_1=5$ mm and $u_2 = 8$ mm. Calculate the displacements at $x = L/4, L/3$ and $L/2$.
11. A steel plate of uniform thickness 25 mm is subjected to a point load of 420 N at mid depth as shown in fig. The plate is also subjected to self-weight. If $E = 2 \times 10^5$ N/mm² and density = 0.8×10^{-4} N/mm³. Calculate the displacement at each nodal point and stresses in each element.
12. An axial load of 4×10^5 N is applied at 30°C to the rod as shown in fig. The temperature is then raised to 60°C. Calculate nodal displacements, stresses in each material and reactions at each nodal point. Take $E_{al} = 0.7 \times 10^5$ N/mm²; $E_{steel} = 2 \times 10^5$ N/mm² ; $\alpha_{al} = 23 \times 10^{-6}$ /°C ; $\alpha_{steel} = 12 \times 10^{-6}$ /°C.
13. Why higher order elements are needed? Determine the shape functions of an eight noded rectangular element.
14. Derive the stiffness matrix for two dimensional truss elements.
15. Determine the slope and deflection of a cantilever beam subjected to a uniformly distributed load q over the entire span and a point load P acting on its free end.
- 16) Consider a steel column subjected to load as shown in following figure. assuming axial loading, determine a) Vertical displacement at all nodes and stress in each element. Take $E=2.5 \times 10^5$ N/mm²; $A=2500$ mm² Divide the column into 4 equal length elements.



17) A steel plate is subjected to an axial load as shown in following figure. Find the approximate deflection and average stress along the plate by using direct formulate method. The plate is 1.5mm thick and has modulus of elasticity $E=2 \times 10^5 \text{ N/mm}^2$.



18) Find a) Nodal displacement b) stress c) Reaction at each node for the bar as shown in following figure



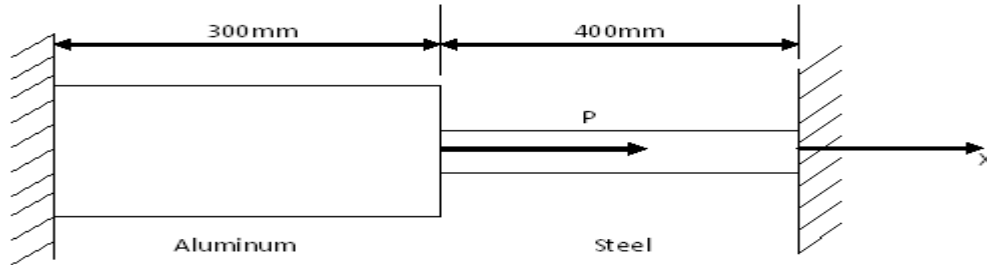
$A= 250\text{mm}^2$;

$E=2 \times 10^5 \text{ N/mm}^2$;

$P=600 \times 10^3 \text{ N}$.

19) Consider the shown in following figure. an axial load $p=200 \times 10^3$ N is applied as shown. Using the penalty approach for handling boundary conditions, find the following:

- Determine the nodal displacements.
- Determine the stress in each material.
- Determine the reaction forces.



$$A_1=2400\text{mm}^2$$

$$E_1=70 \times 10^9$$

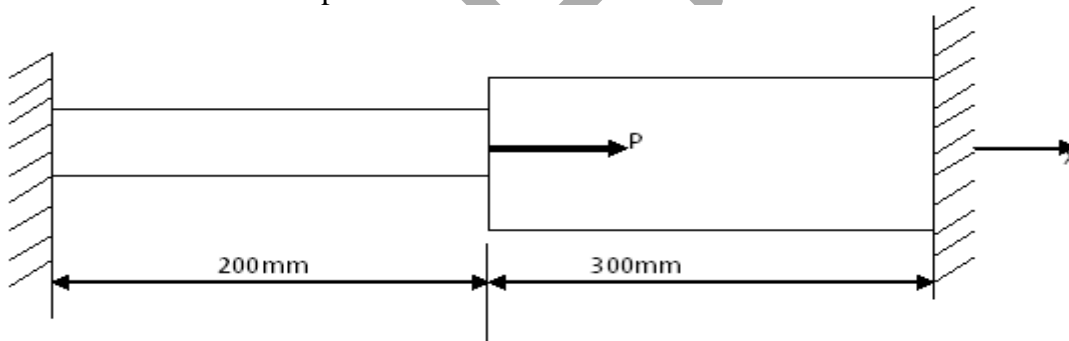
$$A_2=600\text{mm}^2$$

$$E_2=200 \times 10^3\text{N}$$

In following figure, a load $p =60 \times 10^3\text{N}$ is applied as shown. Determine the displacement field, stress and support reactions in the body. Take $E = 20 \times 10^3 \text{ N/mm}^2$.

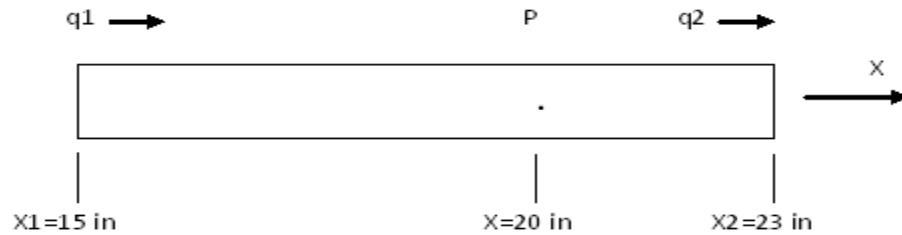
20. An axial load $P= 300 \times 10^3$ N is applied at 20°C to the rod as shown in following figure. the temperature is the Raised to 60°C

- Assemble the stiffness matrix and force vector
- Determine the nodal displacement and element stresses.

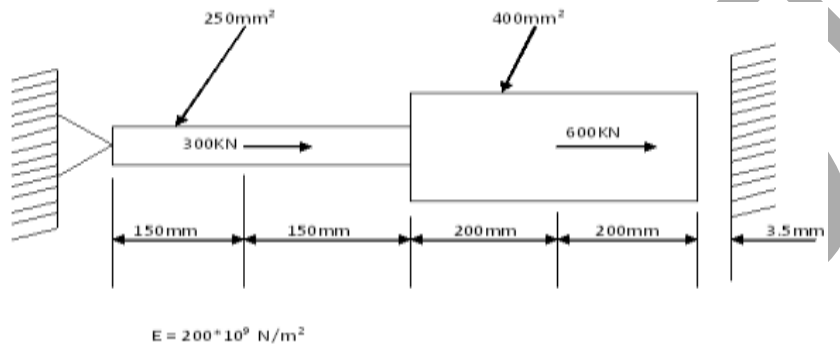


21. Consider the bar in following figure. C/S area $A = 1.2^2$ in and $E = 30 \times 10^6$ psi. If $q_1=0.02$ in and $q_2=0.02$ in. Determine (by manual method):

- The displacement at point P
- The strain ϵ and stress σ
- the element stiffness matrix
- The strain energy in the element



22. Consider the bar in following figure. Determine the nodal displacement, element stresses and support reactions.



UNIT 3- TWO DIMENSIONAL PROBLEMS-SCALAR VARIABLE PROBLEMS

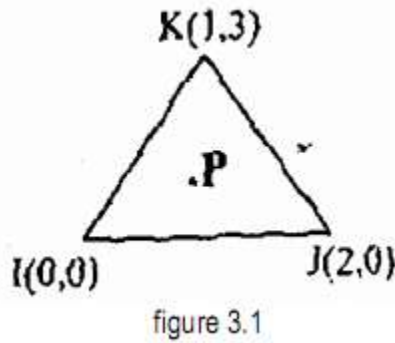
PART A (2 Marks)

1. Write down the shape functions for a 'Rectangular element.
2. State a two dimensional scalar variable problem with an example.
3. What is meant by a CST element? State its properties.
4. In what way a bilinear element is different from simplex and complex element?
5. Define 'Plane stresses and 'Plane strain' with suitable example
6. Differentiate between a CST and LST element
7. What are the differences between uses of linear triangular element?
8. What is meant by a two dimensional vector variable problem?
9. Write down the expression for the stress-strain relationship matrix for a 2-D system.
10. State the expression for stiffness matrix for a bar element subjected to torsion
11. Write down the finite element equation for one-dimensional heat conduction
12. Specify the various elasticity equations.
13. Write down the expression for shape functions for a constant strain triangular element

14. Write a strain displacement matrix for CST element
15. What is LST element?
16. What is QST element?
17. What is meant by plane stress analysis?
18. Write down the stiffness matrix equation for 2-D CST element.
19. Define plane stress analysis.
20. Write a displacement function equation for CST element.

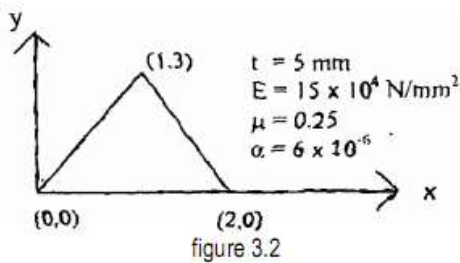
UNIT 3PART B (16 Marks)

1. Find the temperature at a point P(1,1.5) inside a triangular element shown with nodal temperatures given as $T_i = 40^\circ\text{C}$, $T_j = 34^\circ\text{C}$ and $T_k = 46^\circ\text{C}$. Also determine the location of the 42°C contour line for the triangular element shown in figure 3.1.



2. Calculate the force vector for the element
- 3.2. The element

element stiffness matrix and thermal plane stress element shown in figure experiences a rise of 10°C .



3. Derive the stiffness matrix and equations for a CST element.
4. Derive the stiffness matrix and equations for a LST element
5. Derive the shape functions for a bilinear rectangular element .
6. For a 4-noded rectangular element shown in figure 3.6. Determine the temperature at the point (7,4). The nodal values of the temperatures are $T_1 = 42^\circ\text{C}$, $T_2 = 54^\circ\text{C}$ and $T_3 = 56^\circ\text{C}$ and $T_4 = 46^\circ\text{C}$. Also determine the three points on the 50°C contour line.

7. For the plane stress element shown in figure 3.7. Evaluate the stiffness matrix. Assume $E = 210 \times 10^9 \text{ N/mm}^2$, poisson's ratio $\mu=0.25$ and element thickness $t=10\text{mm}$. The coordinates are given in millimeters.

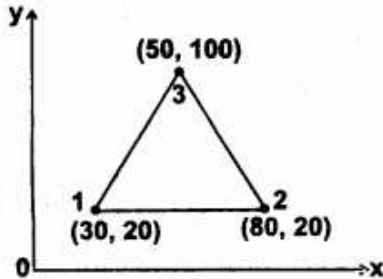


figure 3.7

8. A wall of 0.6 m thickness having thermal conductivity of 1.2 W/mK. The wall is to be insulated with a material of thickness 0.06 m having an average thermal conductivity of 0.3 W/mK. The inner surface temperature is 1000 °C and outside of the insulation is exposed to atmospheric air at 30 °C with heat transfer coefficient of 30 W/m²K. Calculate the nodal temperatures.

9. A furnace wall is made up of three layers, inside layer with thermal conductivity 8.5 W/mK, the middle layer with conductivity 0.25 W/mK, the outer layer with conductivity 0.08 W/mK. The respective thicknesses of the inner, middle and outer layer are 25 cm, 5 cm and 3 cm resp. The inside temperature of the wall is 600 C and outside of the wall is exposed to atmospheric air at 30 °C with heat transfer coefficient of 45 W/m²K. Determine the nodal temperatures .

10. An aluminium alloy fin of 7 mm thick and 50 mm protrudes from a wall, which is maintained at 120 °C. The ambient air temperature is 22 °C. The heat transfer coefficient and thermal conductivity of the fin material are 140 W/m²K and 55 W/mK respectively. Determine the temperature distribution of fin.

11. A steel rod of diameter $d = 2 \text{ cm}$, length $L = 5 \text{ cm}$ and thermal conductivity $k = 50 \text{ W/mC}$ is exposed at one end to a constant temperature of 320 °C. The other end is in ambient air of temperature 20 °C with a convection coefficient of $h = 100 \text{ W/m}^2\text{C}$. Determine the temperature at the midpoint of the rod.

12. Calculate the temperature distribution in a 1-D fin. The fin is rectangular in shape and is 120 mm long, 40 mm wide and 10 mm thick. One end of the fin is fixed and other end is free. Assume

that convection heat loss occurs from the end of the fin. Use 2 elements. The temperature at fixed end is $120\text{ }^{\circ}\text{C}$. Take $k = 0.3\text{ W/mm}^{\circ}\text{C}$; $h = 10^{-3}\text{ W/mm}^2\text{C}$; $T(\text{amb}) = 20\text{ }^{\circ}\text{C}$.

13. A metallic fin, with thermal conductivity $k = 360\text{ W/mc}$, 0.1 cm thick and 10 cm long, extends from a plane wall whose temperature is 235C . Determine the temperature distribution and amount of heat transferred from the fin to the air at 20C with $h = 9\text{ W/m}^2\text{C}$. Take width of the fin to be 1 m .

14. Evaluate the stiffness matrix for the CST element shown in fig. Assume plane stress condition. Take, $t = 20\text{ mm}$, $E = 2 \times 10^5\text{ N/mm}^2$ and $m = 0.25$. The coordinates are given in mm.

15. Assemble the strain-displacement matrix for the CST element shown in fig. Take, $t = 25\text{ mm}$ and $E = 210\text{ Gpa}$.

UNIT 4- TWO DIMENSIONAL PROBLEMS-VECTOR VARIABLE PROBLEMS

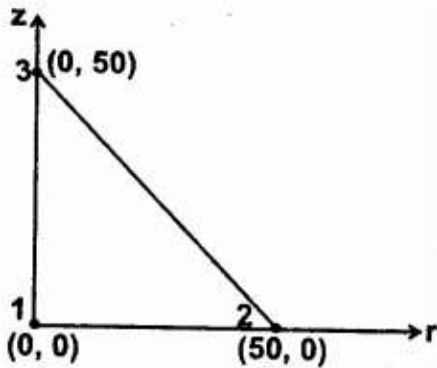
PART A (2 Marks)

1. What are the ways by which a 3-dimensional problem can be reduced to a 2-D problem?
2. What is meant by axisymmetric solid?
3. Write down the expression for shape functions for a axisymmetric triangular element
4. State the conditions to be satisfied in order to use axisymmetric elements
5. State the expression used for 'gradient matrix' for axisymmetric triangular element
6. State the constitutive law for axisymmetric problems.
7. Sketch ring shaped axisymmetric solid formed by a triangular and quadrilateral element
8. Write down the expression for stiffness matrix for an axisymmetric triangular element
9. Distinguish between plane stress, plane strain and axisymmetric analysis in solid Mechanics.
10. Sketch an one-dimensional axisymmetric (shell) element and two-dimensional axisymmetric element.
11. What are the conditions for a problem to be axisymmetric.
12. Give the strain-displacement matrix equation for an axisymmetric triangular element
13. Write down the stress-strain relationship matrix for an axisymmetric triangular element.

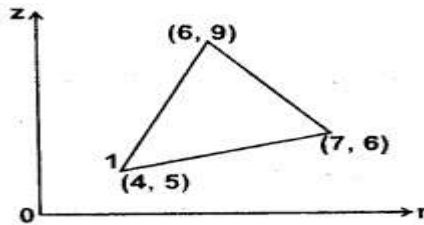
15. What are the ways in which a 3-D problem can be reduced to a 2-D approach.

UNIT 4 PART B (16 Marks)

1. Derive the shape functions for an axisymmetric triangular element
2. Derive an expression for the strain-displacement matrix for an axisymmetric triangular Element.
3. For the axisymmetric element shown in figure, determine the stiffness matrix. Let $E = 2.1 \times 10^5$ MN/m² and $\mu = 0.25$. The coordinates are in mm.



4. Determine the element strains for an axisymmetric triangular element shown in figure 4.4. The nodal displacements are $u_1 = 0.001$, $u_2 = 0.002$, $u_3 = -0.003$, $w_1 = 0.002$, $w_2 = 0.001$, $w_3 = 0.004$. All dimensions are in cm.

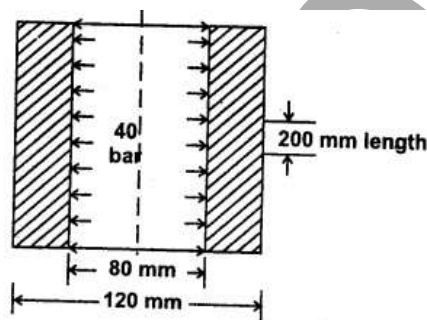


5. The nodal coordinates for an axisymmetric triangular element at its three nodes are $(r_1, z_1) = (30, 10)$, $(r_2, z_2) = (50, 10)$, $(r_3, z_3) = (40, 60)$. Determine the strain displacement matrix for that element.

6. A long hollow cylinder of inside dia 80 mm and outside dia 120 mm is subjected to an internal pressure of 40 bar as shown in figure. By using two elements pin the 20 mm length, calculate the displacements at the inner radius.

7. Derive the expression for the stiffness matrix for an axisymmetric shell element.

8. The nodal co-ordinates for an axisymmetric triangular are given below: $r_1 = 15$ mm, $z_1 = 15$ mm ; $r_2 = 25$ mm , $z_2 = 15$ mm ; $r_3 = 35$ mm , $z_3 = 50$ mm. Determine [B] matrix for that element.



9. Evaluate the temperature force vector for the axisymmetric triangular element shown in the fig. The element experiences a 15°C increase in temperature. Take $\alpha = 10 \cdot 10^{-6}/^\circ\text{C}$, $E = 2 \cdot 10^5$ N/mm², $\nu = 0.25$.

10. Determine the stiffness matrix for the element shown in fig. The co-ordinates are in mm. Take $E = 2 \cdot 10^5$ N/mm² and $\nu = 0.25$.

11. Derive the expression for the element stiffness matrix for an axisymmetric shell element.

12. For the axisymmetric elements shown in fig, determine the element stresses. Let $E = 210$ Gpa and $\nu = 0.25$. The co-ordinates are in mm.

$$u_1 = 0.05 \text{ mm}; w_1 = 0.03 \text{ mm}$$

$$u_2 = 0.02 \text{ mm}; w_2 = 0.02 \text{ mm}$$

$$u_3 = 0 \text{ mm}; w_3 = 0 \text{ mm}$$

13. Derive the strain – displacement matrix [B] for axisymmetric triangular element.

14. Derive the stress-strain relationship matrix [D] for the axisymmetric triangular element.

15. Calculate the element stiffness matrix and the thermal force vector for the axisymmetric triangular element shown in fig. The element experiences a 15°C increase in temperature. The coordinates are in mm. Take $\alpha = 10 \cdot 10^{-6}/^\circ\text{C}$; $E = 2 \cdot 10^5$ N/mm²; $\nu = 0.25$

UNIT 5- ISOPARAMETRIC ELEMENT FORMULATIONS**PART-A (2 Marks)**

1. What is an 'Iso-parametric element'?
2. Differentiate between Isoparametric, super parametric and sub parametric elements.
3. Write down the shape functions for 4-noded linear quadrilateral element using natural coordinate system.
4. What is a 'Jacobian transformation'?
5. What are the advantages of 'Gaussian quadrature' numerical integration for isoparametric elements??
6. How do you calculate the number of Gaussian points in Gaussian quadrature method?
7. Find out the number Gaussian points to be considered for $\int_0^1 (x + 3x - x) dx$
8. What is the Jacobian transformation from a two noded isoparametric element?
9. What is meant by isoparametric formulation?
10. Sketch an general quadrilateral element and an isoparametric quadrilateral element.
11. How do you convert Cartesian coordinates into natural coordinates?
12. Write down the expression for strain-displacement for a four-noded quadrilateral element using natural coordinates.

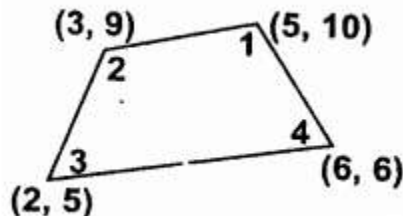
UNIT 5 PART-B (16 Marks)

1. Integrate

$$f(x) = 10 + 20x - (3x^2/10) + (4x^3/100) - (-5x^4/1000) + (6x^5/10000)$$

between 8 and 12. Use Gaussian quadrature rule.

2. Derive the stiffness matrix for a linear isoparametric element.
3. Establish the strain displacement matrix for the linear quadrilateral element as shown in figure at Gauss point $r = 0.57735$ and $s = -0.57735$



4. Write short notes on (a) Uniqueness of mapping of isoparametric elements. (b) Jacobian matrix (c) Gaussian quadrature integration technique

5. (a) Use Gaussian quadrature rule (n=2) to numerically integrate $\int \int xy \, dx \, dy$

(b) Using natural coordinates derive the shape function for a linear quadrilateral element

6. Evaluate the integral using one point and two point Gauss- quadrature. Compare this with exact solution.

$$I = \int_{-1}^1 (3e^x + x^2 + 1/(x+2)) \, dx$$

7. Derive the element stiffness matrix for a linear isoparametric quadrilateral element.

8. Establish the strain – displacement function matrix for the linear quadrilateral element as shown in fig at gauss point $r = 0.57735$ and $s = -0.57735$.

9. The integral $\int_{-1}^1 (r^3 + 2r^2 + 1) \, dr$ can be evaluated exactly by two point gaussian quadrature. Examine the effect on the result if three point integration is applied.

10. For a 4 noded rectangular element shown in fig, determine the following:

Jacobian matrix Strain displacement matrix Element stresses. Take $E = 2 \cdot 10^5 \, \text{N/mm}^2$; $m = 0.25$

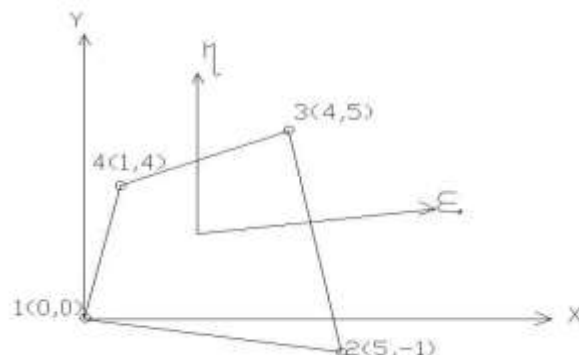
$u = [0, 0, 0.002, 0.003, 0.005, 0.003, 0, 0]$; $\epsilon = 0$; $\eta = 0$. Assume plane stress condition

11. Evaluate the integral $I = \int_{-1}^1 (2 + x + x^2) \, dx$ using gauss quadrature method and compare with exact solution.

12. Evaluate the integral, $I = \int_{-1}^1 \cos \pi x/2 \, dx$ by applying 3 point Gaussian quadrature and compare with exact solution.

13. Evaluate the integral $I = \int_{-1}^1 [3e^x + x^2 + 1/x+2] \, dx$ using one point and two point gauss quadrature. Compare with exact solution.

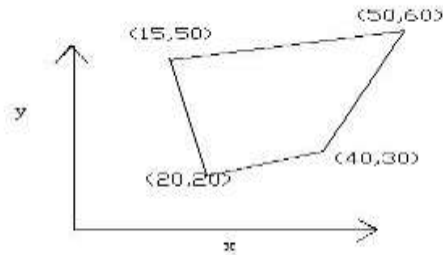
14. For the element shown in following figure determine, the Jacobian matrix



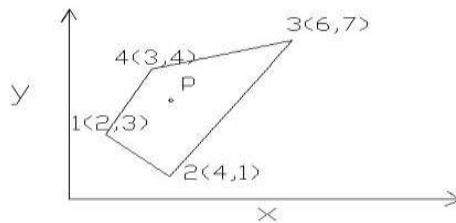
15. Evaluate the following integrals using one point and two point gauss quadrature.

$$I = \int_{-1}^1 [3e^x + x^2 + 1/(x+2)] dx$$

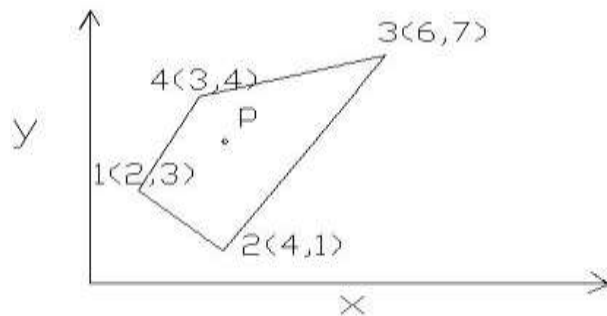
16. A quadrilateral element is shown in fig. Evaluate the Jacobian of transformation.



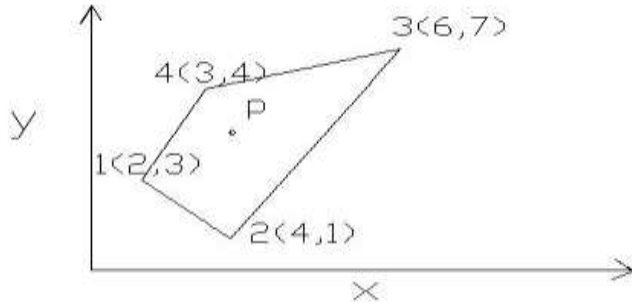
17) Find the Cartesian coordinates of point P. which has natural coordinate values of 0.3 and 0.6 along the axis r and s respectively for the following quadrilateral element.



18) Find the natural coordinate values of P., which has the Cartesian coordinates, value along x and y axis are 4,5 respectively for the following quadrilateral element.



19) Find the Jacobian matrix for the following quadrilateral elements



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